

example of euler path and circuit

example of euler path and circuit are fundamental concepts in graph theory, a branch of discrete mathematics with applications in computer science, engineering, and network analysis. Understanding these examples helps clarify the conditions under which a graph contains an Euler path or an Euler circuit, which are sequences that traverse every edge of a graph exactly once. This article explores the definitions, properties, and practical examples of Euler paths and Euler circuits, emphasizing how these concepts apply to various types of graphs. It also explains the necessary and sufficient conditions for their existence, illustrated through well-known examples. Additionally, the discussion includes the differences between Euler paths and circuits and how they relate to real-world problems such as routing and network traversal. Following this introduction, a detailed table of contents outlines the main sections covered in this comprehensive article.

- Definition and Properties of Euler Path and Euler Circuit
- Conditions for Existence
- Example of Euler Path
- Example of Euler Circuit
- Applications of Euler Paths and Circuits

Definition and Properties of Euler Path and Euler Circuit

An Euler path in a graph is a trail that visits every edge exactly once but does not necessarily start and end at the same vertex. An Euler circuit, also known as an Eulerian circuit or Eulerian cycle, is an Euler path that starts and ends on the same vertex, thus forming a closed loop. These concepts are named after the Swiss mathematician Leonhard Euler, who first studied them in the context of the famous Seven Bridges of Königsberg problem.

Graphs considered in this context are usually undirected, although Eulerian paths and circuits can be defined for directed graphs as well. The significance of Euler paths and circuits lies in their ability to represent efficient traversals over networks, ensuring that every connection or edge is covered exactly once without repetition.

Key properties include:

- An Euler path uses every edge of the graph exactly once.

- An Euler circuit is an Euler path that begins and ends at the same vertex.
- Not all graphs have Euler paths or circuits.
- The degree of vertices plays a crucial role in determining the existence of Euler paths and circuits.

Conditions for Existence

Understanding when Euler paths and Euler circuits exist in a graph is essential. The conditions depend primarily on the degrees of the vertices – that is, the number of edges incident to each vertex.

Conditions for Euler Circuit

A connected graph has an Euler circuit if and only if every vertex has an even degree. This means that the number of edges connected to each vertex is even, allowing for a continuous traversal that returns to the starting point without leaving any edge unvisited.

Conditions for Euler Path

A connected graph has an Euler path but not an Euler circuit if and only if exactly two vertices have an odd degree, and the rest have even degrees. The Euler path will start at one of the vertices with an odd degree and end at the other. If there are more than two vertices of odd degree, no Euler path exists.

Summary of Conditions

- Euler Circuit: All vertices have even degree.
- Euler Path (but no circuit): Exactly two vertices have odd degree.
- No Euler Path or Circuit: More than two vertices have odd degree.

Example of Euler Path

Consider a graph with five vertices labeled A, B, C, D, and E connected as

follows:

- A is connected to B and C.
- B is connected to A, C, and D.
- C is connected to A, B, D, and E.
- D is connected to B, C, and E.
- E is connected to C and D.

Degree of each vertex:

- $\deg(A) = 2$ (edges: AB, AC)
- $\deg(B) = 3$ (edges: AB, BC, BD)
- $\deg(C) = 4$ (edges: AC, BC, CD, CE)
- $\deg(D) = 3$ (edges: BD, CD, DE)
- $\deg(E) = 2$ (edges: CE, DE)

Vertices B and D have odd degrees (3 each), while the others have even degrees. According to the conditions, this graph contains an Euler path but not an Euler circuit. One such Euler path could be:

1. A $\rightarrow$ B
2. B $\rightarrow$ D
3. D $\rightarrow$ C
4. C $\rightarrow$ E
5. E $\rightarrow$ D
6. D $\rightarrow$ B
7. B $\rightarrow$ C
8. C $\rightarrow$ A

This path covers every edge exactly once and begins at vertex A and ends at vertex A or one of the odd-degree vertices depending on the route chosen.

Example of Euler Circuit

Take a simple square graph with vertices W, X, Y, Z connected in a cycle:

- W connected to X and Z.
- X connected to W and Y.
- Y connected to X and Z.
- Z connected to W and Y.

Degree of each vertex:

- $\deg(W) = 2$
- $\deg(X) = 2$
- $\deg(Y) = 2$
- $\deg(Z) = 2$

Since all vertices have even degrees, this graph contains an Euler circuit. An example of an Euler circuit in this graph is:

1. $W \rightarrow X$
2. $X \rightarrow Y$
3. $Y \rightarrow Z$
4. $Z \rightarrow W$

This circuit covers all edges exactly once and ends at the starting vertex W, fulfilling the Euler circuit criteria.

Applications of Euler Paths and Circuits

Euler paths and circuits play a vital role in solving practical problems involving traversal and routing in networks. Their applications extend across various fields including computer science, logistics, and urban planning.

Network Routing

In communication and transportation networks, finding an Euler path or

circuit helps optimize routes that cover every connection without repetition, reducing costs and improving efficiency.

DNA Sequencing

Eulerian paths are used in bioinformatics to reconstruct DNA sequences from short fragments, where the graph edges represent overlaps between fragments.

Postman Problem

The Chinese Postman Problem, a classic optimization problem, seeks the shortest path or circuit covering all edges in a graph, directly related to Euler circuits where the graph is modified to allow such traversal.

Electrical Circuit Design

In circuit design, Euler circuits help ensure that wiring or trace paths cover every required connection without redundancy.

- Efficient route planning in logistics
- Urban street sweeping and garbage collection routing
- Game design and puzzle solving
- Robot path planning in automated systems

Frequently Asked Questions

What is an example of an Euler path?

An example of an Euler path is a path through a graph that visits every edge exactly once but does not necessarily start and end at the same vertex, such as traversing the edges of a graph shaped like a letter 'P' without retracing any edge.

Can you give an example of an Euler circuit?

An Euler circuit is a path in a graph that starts and ends at the same vertex while visiting every edge exactly once. For example, in a square cycle graph with vertices A-B-C-D-A, traversing all edges once and returning to the start forms an Euler circuit.

What is a simple example of a graph with an Euler path but no Euler circuit?

A simple example is a graph shaped like a path with three vertices connected linearly (A-B-C). It has an Euler path starting at A and ending at C but no Euler circuit because it does not start and end at the same vertex.

How does the Königsberg bridge problem illustrate an Euler path?

The Königsberg bridge problem is a classic example where Euler showed that no Euler circuit exists, but an Euler path exists if certain conditions are met. It involves crossing all seven bridges once without repeating, which Euler proved impossible, illustrating the concept of Euler paths and circuits.

What is an example of a graph with an Euler circuit?

A triangle graph with vertices A, B, and C connected in a cycle (A-B, B-C, C-A) has an Euler circuit because you can start at any vertex, traverse each edge exactly once, and return to the starting point.

Can a star-shaped graph have an Euler path or circuit? Provide an example.

A star-shaped graph with a center vertex connected to multiple outer vertices has an Euler path if exactly two vertices have an odd degree. For example, a star with 3 edges has an Euler path but no Euler circuit because the center vertex has degree 3 (odd).

What conditions must a graph meet to have an Euler circuit? Give an example.

A graph must be connected and have all vertices of even degree to have an Euler circuit. For example, a square (4-cycle) graph with vertices connected in a loop satisfies this condition and has an Euler circuit.

Provide an example of a graph that has neither an Euler path nor an Euler circuit.

A graph with more than two vertices of odd degree has neither an Euler path nor an Euler circuit. For instance, a graph with vertices A, B, C, and D where A-B, B-C, and C-D are edges, and vertex B and C have odd degrees without the graph being connected properly to allow a path.

How do Euler paths apply in practical scenarios?

Give an example.

Euler paths are used in routing problems such as garbage collection or street sweeping, where every street (edge) needs to be covered once. For example, planning a route through city streets represented as edges to cover all roads without repetition uses Euler path concepts.

What is a real-world example illustrating an Euler circuit?

A mail carrier delivering mail on a route that starts and ends at the post office and covers every street exactly once exemplifies an Euler circuit if the street network meets the Euler circuit conditions.

Additional Resources

1. *Graph Theory with Applications*

This book provides a comprehensive introduction to graph theory, including detailed discussions on Euler paths and circuits. It offers numerous examples and exercises that illustrate how Eulerian concepts apply to various types of graphs. The clear explanations make it suitable for both beginners and advanced students.

2. *Introduction to Graph Theory*

A classic textbook that covers fundamental topics in graph theory, including Euler paths and circuits. It presents theoretical foundations alongside practical applications, helping readers understand how Eulerian trails are identified and used. The book is well-structured for self-study and classroom use.

3. *Discrete Mathematics and Its Applications*

This widely-used textbook covers a broad range of discrete mathematics topics, with a dedicated section on Euler paths and circuits. It combines theory with real-world examples, demonstrating the significance of Eulerian paths in computer science and network design. The examples are clear and accessible for students.

4. *Graphs, Networks and Algorithms*

Focused on algorithms related to graph theory, this book explores Euler paths and circuits from an algorithmic perspective. It explains how to find Eulerian trails efficiently and discusses their applications in problem-solving. The book is ideal for those interested in computational approaches to graph problems.

5. *Applied Graph Theory*

This text emphasizes practical applications of graph theory, including detailed case studies involving Euler paths and circuits. It explores how these concepts are used in logistics, urban planning, and circuit design. The examples are grounded in real-life scenarios, making the material engaging

and relevant.

6. *Combinatorial Optimization: Algorithms and Complexity*

A rigorous treatment of combinatorial optimization problems, this book includes extensive coverage of Eulerian path problems. It discusses complexity issues and algorithmic solutions related to Euler circuits. The book is suitable for advanced students and professionals interested in optimization techniques.

7. *Introduction to Algorithms*

While broadly focused on algorithms, this book contains key sections on graph traversal techniques, including Euler paths and circuits. It provides pseudocode and explanations for algorithms that detect and construct Eulerian trails. The book is a valuable resource for computer science students and practitioners.

8. *Mathematics of Networks*

This book explores mathematical concepts underlying network theory, with insightful chapters on Eulerian paths and circuits. It examines the role of Euler paths in network connectivity and flow problems. Readers will find a blend of theory and practical applications relevant to telecommunications and transportation.

9. *Fundamentals of Discrete Math for Computer Science*

Designed for computer science students, this book covers essential discrete math topics, including Euler paths and circuits. It offers clear definitions, proofs, and examples that illustrate the importance of Eulerian concepts in computing. The approachable style aids in mastering fundamental graph theory principles.

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